WESLEY COLLEGE

By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2017
QUESTIONS OF REVIEW 6: Integration

Name: ____________________________

Wednesday 9th August | Time: 30 minutes | Mark ~~/28~~ 24

Calculator free.

1. [~~6~~ 5 marks – ~~2~~ 1, 2 and 2]

a) Simplify $\int \frac{2x}{x^2-1}\,dx$

$= \ln|x^2-1| + c$ ✓

b) Express $\frac{2x}{(x-1)^2}$ in the partial fraction form $\frac{A}{(x-1)^2}+\frac{B}{x-1}$

$A + B(x-1)$

~~A(...)...B~~ $= 2x$ ✓

$A = 2 \quad B = 2$

$\therefore \frac{2x}{(x-1)^2} = \frac{2}{(x-1)^2} + \frac{2}{x-1}$ ✓

c) Determine $\int \frac{2x}{(x-1)^2}\,dx$

$= \int 2(x-1)^{-2} + \frac{2}{x-1}\,dx$

$= -2(x-1)^{-1} + 2\ln|x-1| + c$

$= \ln(x-1)^2 - \frac{2}{x-1} + c$

✓ ✓

2. [10 marks – ~~2~~ 1, 3, 3 and ~~2~~ 3]

a) Simplify $\int 12\cos^3 3x \sin 3x \, dx$ by inspection

$$= -\cos^4 3x + C$$ ✓

b) Use the substitution $t = \sin 3x$ to evaluate $\int_0^{\frac{\pi}{6}} 12\cos^3 3x \sin 3x \, dx$

$dt = 3\cos 3x \, dx$

$$= \int_0^1 12(1-t^2).t \, \frac{dt}{3}$$ ✓

$$= \int_0^1 4t - 4t^3 \, dt$$ ✓

$$= 2t^2 - t^4 \Big|_0^1$$

$$= 1$$ ✓

c) Evaluate $\int_1^2 \frac{x}{\sqrt{x-1}} dx$ by using the substitution $t = x - 1$

$$= \int_0^1 \frac{t+1}{\sqrt{t}} dt$$ ✓

$$= \int_0^1 t^{\frac{1}{2}} + t^{-\frac{1}{2}}$$

$$= \frac{2}{3}t^{\frac{3}{2}} + 2t^{\frac{1}{2}} \Big|_0^1$$ ✓

$$= \frac{8}{3}$$ ✓

d) Evaluate $\int_0^{\frac{1}{2}} \tan^2\left(\frac{\pi x}{2}\right) dx$

$$= \int_0^{\frac{1}{2}} \sec^2\left(\frac{\pi x}{2}\right) - 1 \, dx$$ ✓

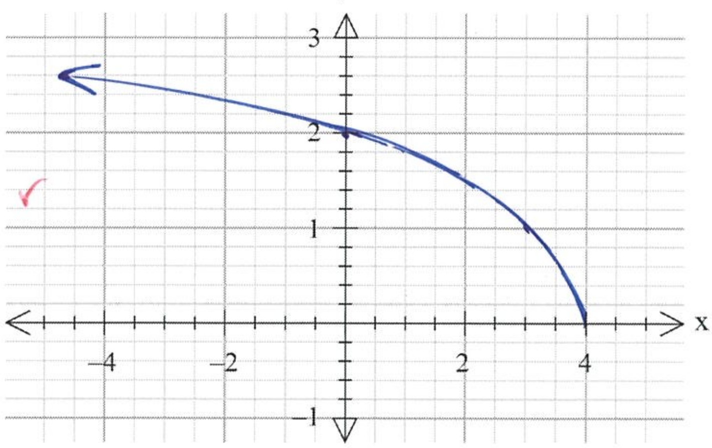
$$= \frac{2}{\pi}.\tan\left(\frac{\pi x}{2}\right) - x \Big|_0^{\frac{1}{2}}$$ ✓

$$= \frac{2}{\pi}\cdot 1 - \frac{1}{2}$$

$$= \frac{2}{\pi} - \frac{1}{2}$$ ✓

3. [~~4~~ 5 marks – 1, 1, 1 and ~~1~~ 2]

a) Draw a quick sketch of
$y=\sqrt{4-x}$



✓

Describe the quantity represented by each of the integrals:

b) $\int_0^3 \sqrt{4-x}\,dx$

area "under" (ie between curve + x axis) $y=\sqrt{4-x}$ between $x=0$ & $x=3$ ✓

c) $2\pi\int_0^4 x\sqrt{4-x}\,dx$

Volume generated by revolving area around y axis
↓
✓ between curve + ~~x~~ axis (0 – 4)

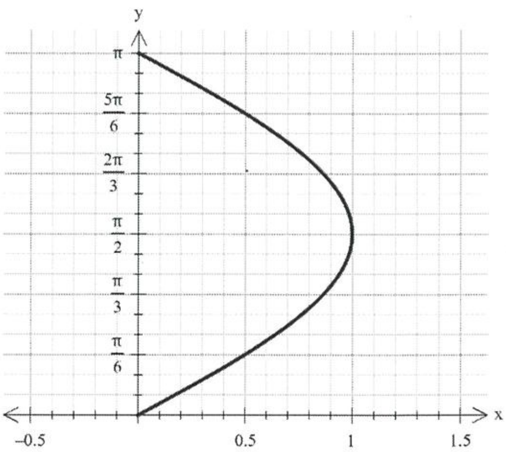
0

d) $\pi\int_0^4 4-(4-x)\,dx$

Volume generated by revolving area between y = 2 ✓ + the curve ✓ around x axis, for $0 \le x \le 4$

4. [4 marks]

What is the volume generated when the curve $x=\sin y$, for $0\le y\le\pi$, is revolved through 360° about the y axis?



$$V_y = \pi\int_0^{\pi} \sin^2 y \; dy \quad ✓$$

$$= \pi\int_0^{\pi} \tfrac{1}{2} - \tfrac{1}{2}\cos 2y \; dy \quad ✓$$

$$= \pi\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)\Big|_0^{\pi} \quad ✓$$

$$= \pi\left(\frac{\pi}{2} - 0\right)$$

$$= \frac{\pi^2}{2} \text{ units}^3 \quad ✓$$